RAND Journal of Economics
Vol. 53, No. 2, Summer 2022
pp. 386–403

Consumer search and optimal information

Mustafa Dogan*

and

Ju Hu**

This article studies an information design problem in a sequential consumer search environment. Consumers, whose valuation of firms' products is uncertain, observe a noisy signal about the valuation upon being matched with a firm. The goal is to characterize those signal structures that maximize consumer surplus. We show that the consumer-optimal signal structure can be found within the class of conditional unit-elastic demand signal distributions. A rich set of properties and comparative statics of the consumer-optimal signal distributions are also derived.

1. Introduction

■ The extensive rise in the availability of information has become a central element transforming markets and the way they function in recent years. Having access to an immense amount of information through various channels, consumers receive significant assistance in choosing between alternatives. This affects their behavior and in turn the behavior of the other market participants, creating market-wide implications.

The channels through which consumers gain access to information include various types of business platforms. Some of them are online review platforms, like Yelp, focusing purely on information provision. Some others are online marketplaces, like eBay, managing the underlying information alongside transactions. Understanding their users' needs and their own business goals, these platforms also carefully design their websites to organize, structure, and label the content to disclose the relevant information. For example, platforms can choose which features of the products to disclose and which elements to emphasize more, at what level of detail to

* MIT Sloan School of Management; mdogan@mit.edu.
** Peking University; juhu@nsd.pku.edu.cn.
We are grateful to the editor, David Martimort, and to two anonymous referees for their comments and suggestions, which significantly improved the article. We are also grateful to Jimmy Chan, Yijuan Chen, Yongmin Chen, Anovia Yifan Dai, Selman Erol, Hanming Fang, Alexandre Jacquillat, Fei Li, Jianpei Li, George Mailath, Hodaka Morita, Ji Shen, Yoichi Sugita, Qianfeng Tang, Cheng Wang, Xi Weng, Zenan Wu, Yongchao Zhang, Jidong Zhou, as well as seminar participants at ITU, PKU, SHUFE, UIBE, the Hitotsubashi-Peking University Second Economics Workshop, the 2018 AMES, the 2018 CMES, the Second CCER Institute, the 2018 International Conference on Economic Theory and Applications at SWUFE, and the 2018 SEA 88th annual conference. We also thank Yu Li and Tan Gan for their excellent research assistance. Hu acknowledges the financial support of the NSFC (Grant No. 71803004). All remaining errors are ours.

provide information, whether to highlight negative reviews or positive reviews about the products, which algorithm to use to detect fake reviews, and so forth. All these aspects of design will shape consumers' perceptions of products and services, which in turn will determine the market interactions and performance, and ultimately the profitability of the platforms. Naturally, each platform chooses the structure of the information in a way to balance the needs of users with the goals of the business.

In this article, we formally investigate a platform's optimal design of its information provision. We consider a platform as a search market where firms compete by setting prices and consumers search sequentially. By incurring a search cost, a consumer can sample a firm, upon which he observes its price and receives a noisy signal about the match quality. The platform designs the information disclosure rule that governs how this noisy signal is generated. Our goal is to understand the information disclosure rules that maximize consumer welfare. This objective is most relevant for online review platforms, such as Yelp and TripAdvisor. These platforms obtain their revenue mostly from advertising. Because advertising revenue is determined by the volume of visitors, who are mainly consumers, these platforms naturally want to provide the most benefits to consumers to attract as many of them as possible.

We adopt the information design approach and focus on information disclosure rules that are independent across firms.[1] The fact that there is no *a priori* restriction on the form of the information disclosure for every firm's product reflects the extensive flexibility that platforms can exploit in their practical design. It can also help us understand the boundary of the welfare effects of information in these markets.

We first show that every equilibrium consumer surplus that can possibly arise under an arbitrary signal distribution can be achieved by a *conditional unit-elastic demand signal distribution*. Such a signal distribution has a simple structure. It incorporates an atom that reveals low match value and a continuum of signals that reveal high match values. In the corresponding equilibrium, consumers actively search in the market, and they reject the equilibrium price offering if and only if the low match value atom signal is observed. The distribution of signals that indicate high match values are tailored to induce a unit-elastic demand over a certain price range for each firm, given the search strategy of the consumers. This form generalizes the unit-elastic demand signal distribution in Roesler and Szentes (2017), and we will discuss the relationships in more detail in Section 3.

The structure of such a signal distribution is very natural as it simultaneously accounts for the two roles that information plays in this search market. On the one hand, the signal distribution informs the consumers whether the current match value is low or high, based on which the consumers decide whether to accept the current offering. This induced search behavior determines the equilibrium expected match quality, the total cost of search, and thus the total welfare. On the other hand, the specially distributed high match value signals guarantee that the firms have no incentive to deviate from a targeted equilibrium price. This, in turn, determines how the total welfare of this market is divided between the consumers and firms.

Restricting attention to conditional unit-elastic demand signal distributions, there is a unique consumer-optimal one. Its determination involves a trade-off between the two roles of information mentioned above. Loosely speaking, higher total welfare requires more information for the consumers to find a better match, but at the same time, more information also leads to higher equilibrium price, as it gives rises to a more differentiated market, which softens the competition.[2] Therefore, unlike the case in Roesler and Szentes (2017) where the consumer-optimal signal distribution always maximizes the total welfare, the optimal one in this search market must balance the total welfare and the equilibrium price.

[1] We rule out the case where the platform reveal information about relative valuations, such as ranking information.

[2] Anderson and Renault (1999) focus on a parameterized class of value distributions, and find that a more differentiated market may result in lower price because it leads to more search, which intensifies the competition among the firms. In the presence of flexible information design, more information (hence more differentiated market) does not necessarily lead to more search, and higher total welfare is always associated with higher minimal feasible equilibrium price.

It is intuitive that less search friction is beneficial for consumers, as it promotes competition among the firms and saves the total cost of searching. Consequently, the optimal consumer surplus is strictly increasing as the search cost decreases. Moreover, we show that the optimal design for a market with less search friction should set a smaller probability of trade. This confirms the intuition that the consumer-optimal signal distribution should make use of less search friction by inducing more searches. We also provide a characterization of the limiting behavior of the consumer-optimal signal distributions as the search cost vanishes.

In addition to analyzing which signal distribution is optimal, we also investigate which signal distribution is never consumer-optimal. That is, it is not optimal regardless of the underlying true value distribution. Intuitively speaking, if a signal distribution induces an equilibrium in which the consumer surplus is too low compared to the total welfare, this signal distribution is never consumer-optimal. A sufficient condition is that the equilibrium profits of a matched firm exceed the search cost of this market. This condition is particularly interesting because it only involves equilibrium quantities that are easy to observe, rather than the details of the underlying signal distribution.

In our baseline model, we only focus on pure strategy equilibria, but we also extend our analysis to show that designing signal distributions that allow mixed strategy equilibria with active search cannot improve consumer welfare. This is because every consumer surplus from a mixed strategy equilibrium must be achieved by a pure strategy equilibrium in a market with a larger search cost. Hence, this consumer surplus, which is weakly lower than the highest level we obtain from a pure strategy equilibria in this new market by definition, is strictly less than the highest level in the original market as our comparative statics results show. We also apply a similar idea to show that asymmetric design for a prominent firm cannot make consumers strictly better off than the simple symmetric design.

□ **Related literature.** Recently, there has been a growing literature on Bayesian persuasion and information design, initiated by Rayo and Segal (2010) and Kamenica and Gentzkow (2011); see Bergemann and Morris (2019) for an extensive survey. Our article is closely related to the work of Roesler and Szentes (2017), who study buyer-optimal information structure in a monopoly pricing setting. They first introduce the notion of the unit-elastic demand signal distribution and show that the maximal buyer surplus can be achieved by a signal distribution within this class.[3] We extend their analysis to a competitive environment. The most important feature of this environment is that the consumers' outside option, which governs the consumers' search incentives and in turn determines the firms' demand curve and pricing incentives, arises endogenously in equilibrium. This nature imposes a significant challenge in how to construct signal distributions that can achieve all possible levels of consumer surplus under arbitrary signal distributions. To deal with such difficulty, we rely on the Rothschild–Stiglitz approach proposed by Gentzkow and Kamenica (2016). It allows us to represent signal distributions by certain convex functions and transforms the daunting feasibility constraints into simpler geometrical conditions. With this approach, we show how to generalize the construction in Roesler and Szentes (2017) to incorporate the consumers' endogenous outside option as part of the design. We discuss this generalization in more detail in Section 3.

More recently, Armstrong and Zhou (2021) also study an information design problem in a competitive environment. Rather than the search market, their main focus is duopolistic competition in a discrete choice model. The major challenge of information design in this environment is that the consumer has multiple outside options and the one from the competing firm is stochastic. By focusing on the case where the consumer always purchases from one of the two firms, the authors show that this difficulty can be overcome as disclosure information about relative

[3] Condorelli and Szentes (2020) extend the analysis to the classical hold-up problem, where there is no fixed prior about the buyer's valuation, and Choi, Kim, and Pease (2019) analyze a search good environment. Both studies find that the unit-elasticity plays an important role in maximizing the buyer's welfare.

valuation is sufficient for the market. Under this observation, Armstrong and Zhou (2021) characterize both the consumer and industry optimal signal distribution, as well as the welfare limits of this market.

Our consumer search model is based on the seminal work by Wolinsky (1986). We introduce an information design problem into this framework. In a similar consumer search framework, Bar-Isaac, Caruana, and Cuñat (2012) analyze a model where firms compete not only in prices, but also their product designs, of which designing information to provide to consumers is one interpretation. By restricting attention to a parameterized family of signal distributions that are ordered by the demand rotation order in Johnson and Myatt (2006), Bar-Isaac, Caruana, and Cuñat (2012) show that every firm provides the maximal or minimal level of information. In contrast, our article considers information design by a third party, for example, a platform, that cares about only consumers, and we do not impose any restrictions on the form of information provision. This allows us to investigate the limit of the welfare effect of information in search markets. As we show, the consumer-optimal information in our framework is not extremal. It optimally trades off providing information to consumers and controlling firms' prices.

There is also a strand of the literature studying decentralized information disclosure by competitive firms within the information design framework. Au and Kawai (2020) abstract away from pricing incentives and analyze competition among firms that disclose their own product information to persuade buyers. Hwang, Kim, and Boleslavsky (2019) consider an oligopoly model in which firms compete not only in prices, but also their advertising strategies about how much product information to provide. Board and Lu (2018) study a search setting in which products across firms are homogeneous and the firms compete in how much information to disclose about the common state, taking into account that the buyer can always move on, at a positive search cost, to a competitor. Au and Whitmeyer (2018) consider a related information design problem in a directed search setting where firms in an oligopolistic market compete in attracting and persuading buyers through their information disclosure about their own products of heterogeneous qualities. Our setting is quite different from these articles, as firms only compete in their prices and product information disclosure is designed by a third party, such as a platform.

The remainder of this article is organized as follows. We introduce the formal model and the information design problem in Section 2. In Section 3, we construct the class of conditional unit-elastic demand signal distributions and show that it is rich enough for us to analyze the consumer-optimal design. In Section 4, we study the determination and properties of the consumer-optimal signal distribution. In Section 5, we discuss two extensions of the baseline model: equilibria in mixed strategies and asymmetric design for a prominent firm. Section 6 concludes and discusses potential future research. The Appendix contains the proof of Proposition 1. All the missing proofs, unless otherwise stated, can be found in the online Appendix.

2. Model

□ **Setup.** We consider a platform as a search market. The framework is based on a model of sequential consumer search due to Wolinsky (1986). We consider its limit case where there are a continuum of risk neutral firms and a continuum of risk neutral consumers.[4] Each firm supplies a single product. The firms' costs of providing their products are normalized to zero. Each consumer wishes to purchase one unit of one product from the market. The value of a firm's product to a consumer is u, which is distributed according to a cumulative distribution function F over [0,1]. Let μ denote the expected value, that is, $\mu \equiv \int_0^1 u\mathrm{d}F(u) \in (0, 1)$.

The market interacts as follows: Firms simultaneously choose a price for their own product. Consumers initially have imperfect information about all prices and match values. They must gather price and value information through a sequential search process. By incurring a search

[4] This setting rules out the case where there are only a finite number of firms and where the consumers' search order is endogenous. Such setting is analyzed in Zhou (2011), Armstrong (2017), and Choi, Dai, and Kim (2018) given exogenously fixed information disclosure.

cost $s \in (0, \mu)$, a consumer can discover a particular product's price, say p, and receive a noisy signal, say q, about the match value.[5] Such signal q is generated by an underlying information technology, which governs how much product information to reveal to a consumer when he is matched with a firm. Throughout the article, we restrict attention to the case where this signal q only reveals information about the current match value and is independent of the match values from other firms. This is mainly for tractability. It rules out some strategies that a platform in practice can adopt, such as providing ranking information.[6] We also assume that the information technology is identical across all matches.[7] Based on the signal, the consumer forms expectation $\mathbb{E}[u|q]$ about the match value and then decides whether to purchase from this currently matched firm. If he purchases, he stops searching and leaves the market. In this case, his expected surplus is $\mathbb{E}[u|q] - p$ and the profits of the firm are p. If he does not purchase, he and the firm get zero from the current match. He then can decide whether to continue searching. As this environment is stationary, whether the consumers have free recall or not does not make a difference.

We are interested in the consumer-optimal information technology that gives consumers the highest equilibrium surplus. As consumers are risk neutral and their purchasing decisions only depend on the conditional mean $\mathbb{E}[u|q]$, the firms' demand and their pricing decisions are entirely driven by the marginal distribution of the conditional mean. Therefore, the problem of designing a consumer-optimal information technology can be reduced to identifying the marginal distribution of the conditional mean that gives consumers the highest equilibrium surplus. It is well known that a distribution of the conditional mean G is induced by some information technology if and only if it is a *mean-preserving contraction* of F: $\int_0^1 q \mathrm{d}G(q) = \int_0^1 q \mathrm{d}F(q)$ and $\int_0^x G(q)\mathrm{d}q \leq \int_0^x F(q)\mathrm{d}q$ for all $x \in [0, 1]$. See, for instance, Blackwell (1951), Gentzkow and Kamenica (2016), Kolotilin (2018), and Dworczak and Martini (2019). We refer to such a distribution G as a *feasible signal distribution* and let $\mathcal{G}_F$ be the set of all feasible signal distributions.[8] For one example, F itself is the most informative signal distribution in $\mathcal{G}_F$. It always perfectly reveals the realized match value to consumers. For another example, F_0, which specifies an atom of size one at μ, is the totally uninformative signal distribution. It reveals no information about the realized match value to the consumers.

□ **Equilibrium.** We focus on symmetric pure strategy equilibria until Section 5, where we extend our analysis to equilibria in mixed strategies. A symmetric pure strategy equilibrium is a price p and a stopping rule for consumers such that (i) the consumers' stopping rule is optimal given that all firms charge p, and (ii) no firm has an incentive to charge a different price given the consumers' stopping rule and the belief that all firms charge p.[9] Every signal distribution induces a trivial equilibrium where all firms charge sufficiently high price so that no consumer participates in the market at all. Such equilibrium leaves zero surplus to the consumers. As our goal is to design a consumer-optimal signal distribution, we focus on equilibria in which consumers actively search and earn positive surplus. We call this kind of equilibrium a *symmetric pure strategy equilibrium with active search*, or simply an equilibrium if no confusion arises.

[5] If $s \geq \mu$, it is optimal for consumers not to search at all as long as firms charge nonnegative prices. Therefore, we rule out this uninteresting case.

[6] This kind of information disclosure rule, that is, disclosure of relative valuation, is extensively studied in Armstrong and Zhou (2021).

[7] We discuss an asymmetric case in Section 5 where the platform adopts different information technology for a prominent firm and non-prominent firms.

[8] Every realized conditional mean q from G can be equivalently viewed as an unbiased signal $\mathbb{E}[u|q] = q$. This explains why we term G as a signal distribution.

[9] We follow the tradition in the consumer search literature and assume that consumers' beliefs are passive: Consumers do not update their beliefs about other firms' prices if a firm deviates.

For any signal distribution $G \in \mathcal{G}_F$, let $c_G : [0, 1] \to \mathbb{R}_+$ be the consumers' *incremental benefit function*, defined as

$$c_G(x) \equiv \int_{[x,1]} (q - x)\mathrm{d}G(q), \quad \forall x \in [0, 1]. \tag{1}$$

This function captures the consumers' search incentives, as the value $c_G(x)$ is each consumer's incremental gain from one more search with a match of expected quality x at hand. By integration by parts, $c_G(x) = \int_x^1 (1 - G(q))\mathrm{d}q$.[10] Thus, the derivative of this function also captures the firms' demand curve $1 - G(x-) = -c'_G(x-)$.[11] The following lemma characterizes an equilibrium in terms of this incremental benefit function.

Lemma 1. Suppose the signal distribution is $G \in \mathcal{G}_F$. There is a symmetric pure strategy equilibrium with active search in which consumer surplus is $v \geq 0$ if and only if there exists a signal cutoff b such that the following two conditions are satisfied:

$$c_G(b) = s, \tag{2}$$

and

$$-c'_G(b-)(b - v) \geq -c'_G(x-)(x - v), \quad \forall x \in [0, 1]. \tag{3}$$

In this equilibrium, each firm charges price $p = b - v$ and consumers purchase at p if and only if they receive a signal greater than or equal to b.

Lemma 1 is a standard result in the consumer search literature and thus its proof is omitted. The idea is the following. When all firms charge the same price, consumers face a stationary environment. As a result, the consumers' optimal stopping rule is a cutoff rule. In particular, consumers stop and purchase from a firm with price p' and signal q if and only if $q - p' \geq v$, where v is consumers' equilibrium surplus. As all firms charge the same price, say p, on the equilibrium path, the consumers' equilibrium signal cutoff should be $b \equiv v + p$. Condition (2) characterizes this signal cutoff b. With signal b at hand, the consumers should be indifferent between stopping and one more round of search, as the incremental benefit of one more search, $c_G(b)$, equals the search cost s.[12] Condition (3) is about firms' incentives. It requires that no firm has an incentive to deviate from the equilibrium price $p = b - v$ to any other price. The left-hand side is a firm's profits from the equilibrium price $p = b - v$. The right-hand side is its profits from deviating to price $p' = x - v$. Facing such a price, the consumers' signal cutoff is $p' + v = x$. Thus, the firm's associated demand and profits are $1 - G(x-) = -c'_G(x-)$ and $-c'_G(x-)(x - v)$, respectively.[13] As there is a one-to-one relationship between the firm's price and consumers' signal cutoff for that price, the firms' pricing decision can be equivalently viewed as a decision to set a cutoff signal. In this respect, an alternative interpretation of condition (3) is that no firm has an incentive to deviate from the equilibrium signal cutoff b.

With a slight abuse of terminology, we call a pair (b, v) that satisfies conditions (2) and (3) an equilibrium. We note that the expected total welfare of equilibrium (b, v) is also b. This is readily seen from the equilibrium identity $b = v + p$, as the right-hand side is the sum of consumer surplus and industry surplus. In later analysis, some results will be more intuitive if we interpret b as the total welfare.

[10] See, for example, Theorem 21.67 in Hewitt and Stromberg (1965) for integration by parts for Lebesgue-Stieltjes integrals.

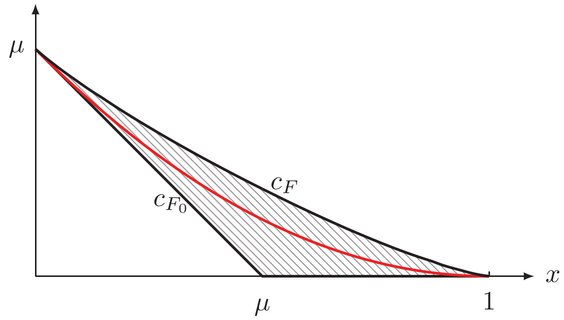
[11] We use $c'_G(x-)$ to denote the left derivative of c_G at $x \in (0, 1]$ and $G(x-)$ to denote the left limit of G at x.

[12] This optimal stopping rule is well known in the search literature. For instance, see Weitzman (1979).

[13] We have implicitly assumed that the consumers always accept the offering when they are indifferent. This assumption is made for ease of exposition. In the online Appendix, we show that relaxing this assumption and allowing consumers to randomize do not change our result at all.

FIGURE 1

FEASIBLE INCREMENTAL BENEFIT FUNCTIONS



□ **Information design problem.** Because Lemma 1 indicates that the effect of a signal distribution on the corresponding equilibrium behavior is encoded in its incremental benefit function, we can formulate our information design problem as choosing a feasible incremental benefit function. Formally, let $\mathcal{C}_F \equiv \{c_G \mid G \in \mathcal{G}_F\}$ be the set of all feasible incremental benefit functions. Then, the problem of designing a consumer-optimal signal distribution can be formulated as

$$\begin{aligned} &\max_{c \in \mathcal{C}_F} v \\ &\text{s.t.} \quad \exists\, b \text{ such that } (b, v) \text{ and } c \text{ satisfy (2) and (3).} \end{aligned} \tag{4}$$

It is possible that no incremental benefit function $c \in \mathcal{C}_F$ induces an equilibrium with active search. That is, the constraint set of (4) may be empty.

If, instead, there is at least one incremental benefit function $c \in \mathcal{C}_F$ that induces such an equilibrium, we say the *search market* (F, s) *admits active search*. We will focus on these search markets in the following analysis. Corollary 1 in Section 3 provides a characterization of when a search market admits active search.

3. Conditional unit-elastic demand signal distributions

■ In this section, we construct a special class of feasible signal distributions, which will be called conditional unit-elastic demand signal distributions. This class is rich enough so that we can restrict our attention to it to find a consumer-optimal one.

□ **Feasible incremental benefit functions.** Let c_{F_0} be the incremental benefit function under the totally uninformative signal distribution F_0. It is easy to see that $c_{F_0}(x) = \max\{\mu - x, 0\}$. Let c_F be the incremental benefit function under full information. Both c_{F_0} and c_F are elements in $\mathcal{C}_F$. The following lemma, which builds on Gentzkow and Kamenica (2016), shows that the set of all feasible incremental benefit functions is just the set of all convex functions bounded between c_{F_0} and c_F.[14]

Lemma 2. A function $c : [0, 1] \to \mathbb{R}$ is the incremental benefit function of a feasible signal distribution if and only if it is convex and $c_{F_0} \le c \le c_F$. Therefore, $\mathcal{C}_F = \{\text{convex } c : [0, 1] \to \mathbb{R} | c_{F_0} \le c \le c_F\}$.

Figure 1 gives an illustration of $\mathcal{C}_F$ based on Lemma 2. The lower black curve is c_{F_0}. It is a downward-sloping 45-degree line over $[0, \mu]$ and coincides with the x-axis over $[\mu, 1]$. The higher black curve is c_F. The shaded area between c_{F_0} and c_F represents the range of $\mathcal{C}_F$. Any

[14] Because it is a straightforward variation of Proposition 1 in Gentzkow and Kamenica (2016), its proof is omitted.

convex function in this area is a feasible incremental benefit function, and vice versa. The condition $c_{F_0} \leq c \leq c_F$ has an intuitive interpretation: More information helps the consumers make better decisions on keeping the original match at hand or taking the new match arising from one more round of search. The red curve is an example of a feasible incremental benefit function.

Given any feasible incremental benefit function $c \in \mathcal{C}_F$, we can deduce the underlying signal distribution G by $G(x) = 1 + c'(x+)$. This formula, together with $G(x-) = 1 + c'(x-)$, allows us to infer two qualitative features of G directly from the graph of c, which will be helpful in understanding the following analysis. First, if the incremental benefit function c has a kink, that is, is not differentiable, at some point $x \in [0, 1]$, then G has an atom at x. This is because $G(x) - G(x-) = c'(x+) - c'(x-) > 0$.[15] Second, if c is a straight line segment over some interval $[x', x'']$, then the signals in (x', x'') have zero probability under G. This is because $G(x) = 1 + c'(x+)$ is a constant over (x', x''). The graph of c_{F_0} in Figure 1 provides an example of both features. Because c_{F_0} is a straight line segment over $[0, \mu]$ and $[\mu, 1]$, F_0 places no mass over $(0, \mu)$ and $(\mu, 1)$. Because $c'(0+) = -1$ and $c'(1-) = 0$, neither $x = 0$ nor $x = 1$ is an atom of F_0. Thus, F_0 places all the probability mass at the single point $x = \mu$, at which c_{F_0} has a kink, implying that F_0 is the totally uninformative signal distribution.

□ **Construction.** We now construct a parametric class of signal distributions in terms of their incremental benefit functions. Each signal distribution is characterized by three parameters a, b, and v, and induces equilibrium (b, v). We will also show that every equilibrium consumer surplus that can arise under a feasible signal distribution can be attained by a signal distribution in this class.

Let $\bar{b}$ be the value satisfying $c_F(\bar{b}) = s$. For any $a \in [0, \mu - s)$, $b \in [\mu - s, \bar{b}]$ and $v \in [a, b)$, define

$$c_{a,b,v}(x) \equiv \begin{cases} \mu - x, & \text{if } x \in [0, a], \\ s - \rho(x - b), & \text{if } x \in (a, b], \\ \max\left\{s - \pi \log \frac{x-v}{b-v},\ 0\right\}, & \text{if } x \in (b, 1], \end{cases} \tag{5}$$

where $\rho \equiv \frac{\mu-s-a}{b-a}$ and $\pi \equiv \rho(b - v)$. Figure 2 illustrates a typical $c_{a,b,v}$ in panel (a) and its underlying signal distribution in panel (b). Over $[0, a]$, $c_{a,b,v}$ coincides with c_{F_0} (the downward-sloping 45-degree line). Then, it becomes the straight line segment that connects the points $(a, \mu - a)$ and (b, s). The corresponding slope is just $-\rho = -\frac{\mu-s-a}{b-a}$. This construction of $c_{a,b,v}$ over $[0, b]$ creates a kink at a, which corresponds to an atom signal in the signal distribution. Because this signal a is the only signal below b, we refer to it as the *low match value atom*.[16] Over $[b, 1]$, $c_{a,b,v}$ takes a particular functional form, which leads to a continuum of signals and potentially an atom ($\bar{x}$ in Figure 2) above b. These signals are distributed in a specific way to ensure that the firms' pricing incentives are satisfied, as we shall see in Proposition 1.

It is easy to see that $c_{a,b,v}$ is convex and satisfies $c_{a,b,v} \geq c_{F_0}$ by construction. Hence, $c_{a,b,v}$ is feasible if and only if $c_{a,b,v} \leq c_F$. Let $\mathcal{U} \subseteq \mathcal{C}_F$ be the set of all feasible $c_{a,b,v}$'s.[17] The following proposition states that it is without loss of generality to restrict attention to $\mathcal{U}$ to find a consumer-optimal signal distribution.

Proposition 1. Every $c_{a,b,v} \in \mathcal{U}$ induces equilibrium (b, v). Conversely, if $c \in \mathcal{C}_F$ induces equilibrium (b, v), then there exists a' and b' such that $c_{a',b',v} \in \mathcal{U}$.

The first part of Proposition 1 comes directly from the way $c_{a,b,v}$ is constructed. Because $c_{a,b,v}(b) = s$ by construction, the consumers' search incentive condition (2) is satisfied. To see that

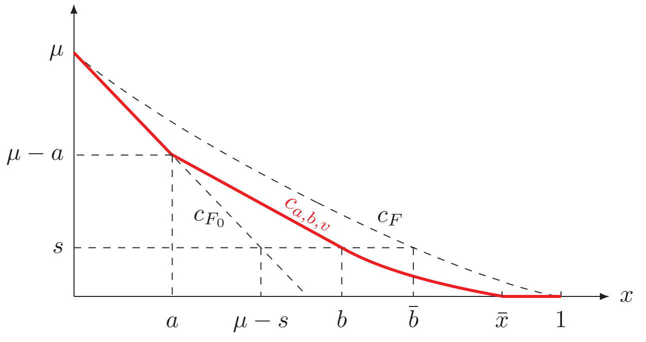
[15] Define $c'(0-) = -1$ and $c'(1+) = 0$.

[16] If $b = \mu - s$, this kink disappears. In this case, the value of a is irrelevant and it is no longer an atom.

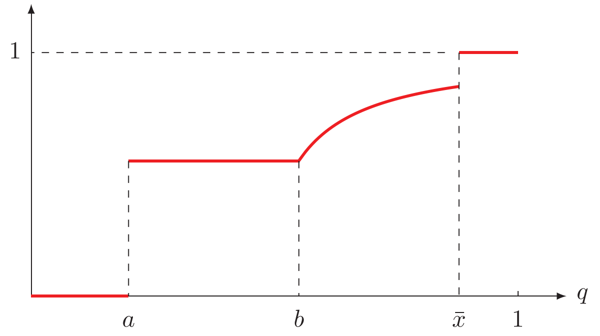
[17] The set $\mathcal{U}$ may be empty. As we will see in Corollary 1, $\mathcal{U} \neq \emptyset$ if and only if the search market (F, s) admits active search.

FIGURE 2

ILLUSTRATION OF CONDITIONAL UNIT-ELASTIC DEMAND SIGNAL DISTRIBUTIONS



(a) Constructed incremental benefit function $c_{a,b,v}$



(b) Conditional unit-elastic demand signal distribution

the firms' pricing incentive constraints are also satisfied, we can divide the constraints in (3) into two groups: *downward* and *upward* incentive constraints. The downward incentive constraints involve deviation to those cutoffs below b. They ensure that no firm has an incentive to charge a lower price than $p = b - v$. These incentive constraints are satisfied due to the crucial restriction $v \geq a$: deviating to a cutoff $x \leq a$ involves charging a non-positive price $x - v$ and deviating to a cutoff $x \in (a, b)$ would not change the demand, that is, $-c'_{a,b,v}(x-) = -c'_{a,b,v}(b-)$.

The upward incentive constraints involve deviation to those cutoffs above b. They ensure that no firm has an incentive to charge a higher price than $p = b - v$. These constraints are satisfied because each firm faces a demand curve with unit elasticity. To see this, observe that each firm's demand from cutoff $x \in (b, \bar{x}]$ is $-c'_{a,b,v}(x-) = \frac{\pi}{x-v}$, where π by definition equals the equilibrium expected profits: $-c'_{a,b,v}(b-)(b - v)$. Therefore, $-c'_{a,b,v}(x-)(x - v) = -c'_{a,b,v}(b-)(b - v)$ for $x \in [b, \bar{x}]$, and firms are in fact indifferent between any cutoffs in $[b, \bar{x}]$. No firm has an incentive to deviate to cutoff $x > \bar{x}$ either, as the demand would be zero.

In the equilibrium, firms charge price $b - v$, and consumers keep searching if and only if they observe the low match-value atom a on the equilibrium path. The corresponding probability of trade per match is just ρ, and the expected profits of a matched firm are just π.

The second part of Proposition 1 asserts that $\mathcal{U}$ is rich enough to induce every equilibrium consumer surplus that can arise under an arbitrary signal distribution. As a result, to find a consumer-optimal one, we only need to focus on the incremental benefit functions in $\mathcal{U}$. To show this result, we show that if (b, v) is an equilibrium under $c \in \mathcal{C}_F$, then either $c_{a,b,v}$ is feasible (if $a \equiv \mathbb{E}[q|q < b] \leq v$), or $c_{0,\mu-s,v}$ is feasible (if $a > v$).[18]

As a direct corollary of Proposition 1, we know that the search market (F, s) admits active search if and only if $\mathcal{U} \neq \emptyset$. Intuitively, this is the case when the search cost is not too high.

Corollary 1. There exists a threshold $s^* \in (0, \mu)$ such that (F, s) admits active search if and only if $s \in (0, s^*]$.

A search market with differentiated products can be active under properly designed information, provided the search friction is small. However, when the search friction is too large, consumers will never participate regardless of the information available to them and the market completely shuts down. Clearly, the threshold s^* increases in F with respect to the mean preserving spread order. This is simply because there is more room to design information in a market with more differentiated products.

□ **Discussion.** In a monopoly pricing setting, Roesler and Szentes (2017) find that the consumer surplus is maximized by a unit-elastic demand signal distribution. Under this signal distribution, the monopolist is indifferent between prices in the support of this distribution, and trade occurs with probability one in the consumer-optimal equilibrium. The signal distribution underlying $c_{a,b,v}$ in (5) extends this idea to the current setting, as the firms are also indifferent between prices over a certain range in equilibrium, but there are two important generalizations.

First, the consumers' outside option, which governs their equilibrium search behavior and the competition between the firms, is endogenously determined in equilibrium. Because equilibrium is a result of the information design, it is intuitive that we should incorporate this outside option as part of the construction of $c_{a,b,v}$. This explains why the consumer surplus v, which coincides with the consumers' outside option in this stationary environment, is an explicit parameter. Second, when $b > \mu - s$, the signal structure $c_{a,b,v}$ specifies an atom a below the equilibrium signal cutoff. In equilibrium, consumers do not purchase from the currently matched firm upon observing this signal, even if the current trade generates positive surplus. This low match value atom is a necessary component that reflects the dynamic nature of our environment. It serves as a way to reveal information about low match qualities and incentivize the consumers to keep searching for a better match.

Because the fact that firms face a unit-elastic demand strictly relies on the consumers' endogenous equilibrium search behavior, we refer to the underlying signal distribution that gives rise to such an incremental benefit function $c_{a,b,v}$ as a *conditional unit-elastic demand signal distribution*.

4. Consumer-optimal design

■ In this section, we analyze the consumer-optimal conditional unit-elastic demand signal distribution and its properties.

□ **Consumer-optimal signal distribution.** By Proposition 1, a consumer-optimal signal distribution solves the following problem:

$$\max_{c_{a,b,v} \in \mathcal{U}} v. \tag{6}$$

Proposition 2 establishes the existence and uniqueness of the optimal solution.

Proposition 2. Suppose $\mathcal{U} \neq \emptyset$. Then, there is a unique consumer-optimal conditional unit-elastic demand signal distribution.

[18] The expectation $\mathbb{E}[q|q < b]$ is with respect to the underlying signal distribution of c.

One way to understand the determination of the optimal signal distribution is to rewrite (6) equivalently as

$$\max_{b\in[\mu-s,\,\bar{b}]}\ \max_{a\in[0,\,\mu-s)}\ \max_{v\in[a,b)} v \quad \text{subject to } c_{a,b,v} \le c_F.$$

Using the equilibrium identity $v = b - p$, we can further transform it into the form

$$\max_{b\in[\mu-s,\,\bar{b}]} \left(b - \min_{a\in[0,\,\mu-s]}\ \min_{p\in(0,\,b-a]} p\right) \quad \text{subject to } c_{a,b,b-p} \le c_F.$$

This problem has a very intuitive interpretation. For every b, $\min_a \min_p p$ subject to the feasibility constraint is the *minimal feasible price* for total welfare b. Although the low match value atom a does not directly appear in the objective function, its role is to determine the equilibrium probability of trade $\rho = \frac{\mu-s-a}{b-a}$, which in turn affects the feasibility constraint in determining the minimal price. Then, $b - \min_a \min_p p$ corresponds to the highest possible consumer surplus given total welfare b. Among all the feasible levels of total welfare, the designer then chooses the optimal one that maximizes the consumer surplus.

In the monopoly pricing setting, Roesler and Szentes (2017) show that the consumer-optimal signal distribution also maximizes the total welfare, but this is no longer the case in the current setting. Intuitively, higher total welfare is associated with providing more information to the consumers to find a better match, but more information disclosure also makes the market more differentiated, which softens the price competition among the firms. More formally, we can indeed show that the minimal feasible price is increasing with the total welfare.[19] Therefore, the optimal choice of b must balance between the total welfare and the price. A careful analysis of how this trade-off changes with the search cost leads to the following intuitive comparative statics result. For search cost s, let $a^*(s)$, $b^*(s)$, $v^*(s)$, and $\rho^*(s)$ be the corresponding consumer-optimal choices of the low match value atom, equilibrium signal cutoff, consumer surplus, and probability of trade, respectively.

Proposition 3. Assume $0 < \underline{s} < \bar{s} < s^*$.

(i) The optimal consumer surplus is strictly higher in markets with smaller search cost: $v^*(\underline{s}) > v^*(\bar{s})$.
(ii) The consumer-optimal signal distributions satisfy: $b^*(\underline{s}) > b^*(\bar{s})$, $a^*(\underline{s}) \ge a^*(\bar{s})$, and $\rho^*(\underline{s}) \le \rho^*(\bar{s})$.
(iii) As the search cost vanishes, $\lim_{s\downarrow 0} v^*(s) = \lim_{s\downarrow 0} b^*(s) = 1$, $\lim_{s\downarrow 0} a^*(s) = \mathbb{E}[q|q < 1]$, and $\lim_{s\downarrow 0} \rho^*(s) = 1 - F(1-)$.

Part (i) of Proposition 3 is about optimal consumer surplus. A smaller search cost is always beneficial to consumers, as it lowers the cost involved in searching and promotes competition among the firms. Consequently, the optimal consumer surplus strictly increases as the search cost decreases. This simple observation plays a central role in Section 5 when we extend our analysis to equilibria in mixed strategies and asymmetric design. Part (ii) describes how the optimal signal distribution changes as the search cost changes. The results are mainly driven by the intuitive fact that if a smaller probability of trade is better than a larger one for consumers in the market with high search cost, then the same comparison between these two probabilities of trade holds in the market with low search cost.[20] Therefore, it is optimal for the designer to induce a smaller probability of trade in a market with smaller search cost. Doing so also leads to higher total welfare and low match value atom.

Part (iii) characterizes the limiting behavior of the optimal consumer surplus and the corresponding optimal signal distribution as the search cost vanishes. In an almost frictionless market, consumers' search incentives and the competition among the firms will be strong enough

[19] See Footnote 1 in the online Appendix for an explanation.

[20] See Claim C.2 in the online Appendix.

so that the consumer-optimal signal distribution can virtually guarantee the first-best outcome to the consumers. Moreover, because the optimal equilibrium signal cutoff $b^*(s)$ approaches the best possible match, eventually all the match values below the best one will be summarized by $a^*(s)$. Thus, in the limit, $a^*(s)$ approaches $\mathbb{E}[q|q < 1]$. Consequently, the corresponding optimal signal distribution converges to either the totally uninformative distribution F_0 if the best match $q = 1$ is not an atom of F (equivalently, $F(1-) = 1$), or the binary distribution $(1 - F(1-)) \circ \mathbb{E}[q|q < 1] + F(1-) \circ 1$ if $q = 1$ is an atom.

□ **Never consumer-optimal signal distribution.** Characterizing the consumer-optimal signal distribution is rather a complicated problem in general. This is mainly because the optimization problem (6) involves infinitely many nontrivial constraints despite its parametric nature with only three choice variables. A more tractable and perhaps more practical question is whether we can tell that a given signal distribution is *not* consumer-optimal. Knowing that a signal distribution is not consumer-optimal at least tells us that it can be improved upon.

More specifically, fix a search cost $s > 0$. We say a signal distribution G is *never consumer-optimal* if G is not consumer-optimal for all possible true value distributions under which G is feasible. Hence, if this G is the signal distribution in a certain market, we immediately know that there can be better information disclosure for the consumers, even though we may not have any knowledge about the true value distribution of this market. Equivalently, a signal distribution G is never consumer-optimal if and only if, in the market where G itself is the true value distribution, full information disclosure is not consumer-optimal. The following proposition provides simple sufficient conditions for a signal distribution to be never consumer-optimal.

Proposition 4. Let G be a signal distribution. Suppose (b, v) is an equilibrium with active search under G and $b > \mu - s$. Let $a \equiv \mathbb{E}[q|q < b]$ be the conditional mean of the signals below the equilibrium signal cutoff. If

$$v < b - \frac{b - a}{\mu - a} s, \tag{7}$$

then G is never consumer-optimal. In particular, if

$$\pi \geq s, \tag{8}$$

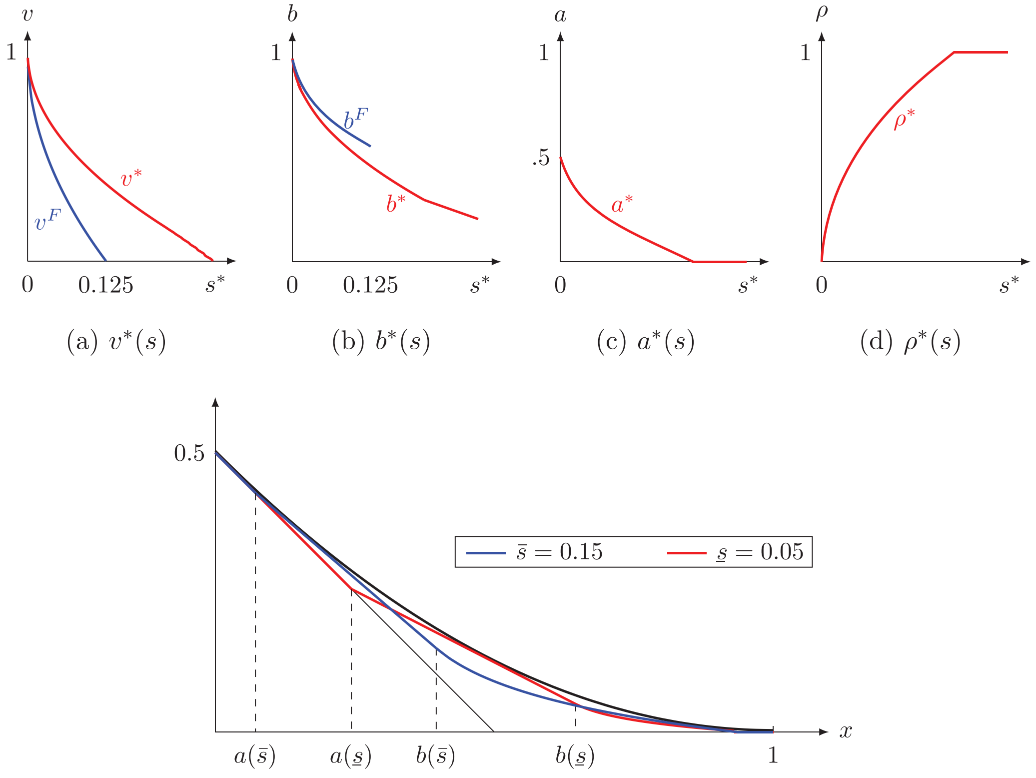
then G is never consumer-optimal, where π is the equilibrium expected profits of a matched firm.

Loosely speaking, if a signal distribution G induces an equilibrium in which the consumer surplus is too low compared to the total welfare, that is, condition (7), then G is never consumer-optimal. In this case, we can find a feasible signal distribution that leads to lower total welfare but strictly higher consumer surplus. Condition (8) is an interesting sufficient condition for (7). As long as the equilibrium expected profits of a matched firm are greater than or equal to the search cost, then G is never consumer-optimal. The most remarkable feature of this surprisingly simple condition is that it does not involve the details of the signal distribution. Rather, it only relies on quantities that are relatively easy to observe and measure: equilibrium price, probability of trade per match, and search cost. If (8) is observed to be true in a real market, we immediately know that the way this market discloses quality information can be improved in terms of consumer surplus.

□ **Example.** We use the uniform value distribution $F \sim U[0, 1]$ to illustrate the previous results. For this value distribution, the threshold for active search is $s^* \approx 0.30$. The red curves in panels (a)–(d) in Figure 3 plot the parameters of the consumer-optimal conditional unit-elastic demand signal distribution as functions of $s \in (0, s^*)$. As seen, these functions are monotonic in the directions described in Proposition 3. As search cost decreases to 0, the optimal consumer surplus and the signal cutoff increase to the best possible match, and the probability of trade goes to zero. Consequently, the low match value atom converges to the mean $\mu = 0.5$. We also see

FIGURE 3

OPTIMAL SIGNAL DISTRIBUTIONS FOR THE UNIFORM VALUE DISTRIBUTION



(e) Optimal signal distribution for $\underline{s} = 0.05$ and $\bar{s} = 0.15$

that the probability of trade becomes 1 when the search cost is sufficiently large. This induces immediate purchase in equilibrium, because it is too costly for the consumers to search.

Panel (e) illustrates the corresponding consumer-optimal incremental benefit functions for search costs $\underline{s} = 0.05$ and $\bar{s} = 0.15$. The fact that these two curves intersect each other indicates that the optimal signal distributions for different search costs in general are not ranked according to the mean-preserving contraction order.

For the uniform distribution, full information disclosure also induces a unique equilibrium, provided $s \leq 0.125$. This equilibrium is characterized by total welfare (signal cutoff) $b^F(s) = 1 - \sqrt{2s}$, price $p^F(s) = \sqrt{2s}$, and consumer surplus $v^F(s) = 1 - 2\sqrt{2s}$. The blue curves in panels (a) and (b) illustrate v^F and b^F, respectively. As we can see, $v^F(s) < v^*(s)$ for all $s \in (0, 0.125]$. This is precisely because (8) holds under full information disclosure and thus uniform F is never consumer-optimal,[21] but when the search cost vanishes, the gap between v^* and v^F diminishes. This is also intuitive. Because full information disclosure always leads to the highest possible total welfare, the only reason why optimal information makes the consumers strictly better off is because it induces a lower equilibrium price, but when the search cost is sufficiently small, the competition among the firms becomes increasingly fierce so that the equilibrium price is

[21] Under full information disclosure, the equilibrium probability of trade is $\rho^F(s) = 1 - b^F(s) = \sqrt{2s}$ and each firm's expected profits are $\rho^F(s) \times p^F(s) = 2s > s$.

already very low even under full information. As a result, there is not much room for further improvement, and the advantage of information design completely disappears in the limit.[22]

5. Extensions

■ In this section, we provide two extensions to our analysis. We first consider mixed strategy equilibria. Then, we consider the possibility of asymmetric design for a prominent firm. As we shall see, neither of these extensions can improve the consumers' welfare.

□ **Equilibria in mixed strategies.** So far, we have only considered pure strategy equilibria. As mentioned in Section 2, this restriction automatically rules out some signal distributions that can induce active search only in mixed strategies. Nonetheless, we show here that this restriction is immaterial to the optimal consumer surplus. No consumer surplus achieved by a mixed strategy equilibrium under a feasible signal distribution can exceed $v^*(s)$—the optimal consumer surplus from pure strategy equilibria.

We continue to focus on the equilibria in which all the firms follow the same pricing strategy. As a result, consumers' optimal stopping decision is still a cutoff rule.[23] Formally, a mixed strategy equilibrium with active search under signal distribution G can be characterized by a pair (σ, v), where σ is the firms' mixed strategy over equilibrium signal cutoffs and $v \geq 0$ is the consumer surplus as before. Similarly as in Lemma 1, (σ, v) is an equilibrium if

$$\int_{\mathrm{supp}(\sigma)} c_G(b)\mathrm{d}\sigma(b) = s, \tag{9}$$

and, for all $b \in \mathrm{supp}(\sigma)$,

$$-(b-v)c'_G(b-) \geq -(x-v)c'_G(x-), \ \ \forall x \in [v, 1], \tag{10}$$

where $\mathrm{supp}(\sigma)$ is the support of σ. Condition (9) is a variant of condition (2). It states that the *average incremental gain* from one more search is equal to the cost of one more search.[24] Condition (10) states that every equilibrium signal cutoff $b \in \mathrm{supp}(\sigma)$ must be optimal for the firms. For each such b, condition (10) takes exactly the same form as condition (3). This is because every firm is still competing with the consumers' outside option v. When the support of σ is a singleton, (σ, v) degenerates into a pure strategy equilibrium.

The following result states that designing signal distributions that induce equilibria in mixed strategies can only make consumers strictly worse off compared to the optimal one among those that induce pure strategy equilibria.

Proposition 5. Suppose (σ, v) is a mixed strategy equilibrium under feasible signal distribution G in the market with search cost s. If $\mathrm{supp}(\sigma)$ is not a singleton, then $v < v^*(s)$.

The logic behind Proposition 5 is simple. If consumer surplus v is achieved by a mixed strategy equilibrium in a market with search cost s, it must be achievable by a *pure strategy equilibrium in a market with search cost greater than s*. To see this, observe that the consumers' incentive constraint (9) implies that there exists an equilibrium cutoff $\tilde{b} \in \mathrm{supp}(\sigma)$ such that

[22] This discussion holds for general value distribution which has a positive continuous density and an increasing hazard rate. We can show that (8) holds for such value distributions under full information disclosure. We thank an anonymous referee for pointing this out.

[23] As before, we continue to assume that the consumers accept the current offering when they are indifferent, for ease of exposition.

[24] Given the firms' mixed strategy σ_p over prices, consumers' optimality condition requires

$$v = -s + \int_{\mathrm{supp}(\sigma_p)} \int_{[0,1]} \max\{q-p, v\}\mathrm{d}G(q)\mathrm{d}\sigma_p(p).$$

Condition (9) is obtained by subtracting both sides by v and letting σ be the distribution of $p + v$.

$c_G(\tilde{b}) > s$. The firms' incentive constraint (10) for $\tilde{b}$ then implies that $(\tilde{b}, v)$ is an equilibrium under G *in the market with search cost* $\tilde{s} \equiv c_G(\tilde{b}) > s$. Thus, v cannot exceed $v^*(\tilde{s})$, but this in turn implies that $v < v^*(s)$ by Proposition 3, as smaller search cost leads to strictly higher optimal consumer surplus from pure strategy equilibria.

□ **Asymmetric design for a prominent firm.** Throughout the previous analysis, we have focused on symmetric information disclosure. All firms face the same information technology and their signal distributions are identical. In principle, a platform does not face such a restriction. It can design different webpage layouts and impose different information disclosure criteria for different firms. We explore this possibility by considering the situation where one firm is more prominent than the others and all consumers match with this firm first, and where the designer can adopt asymmetric designs for the prominent and non-prominent firms.

Prominence is a very natural phenomenon in the online world. For example, platforms can always place one particular firm at the top of the page when displaying the results from its search engine, and consumers are likely to examine the product from that firm first. Under a fixed signal distribution, for example, full information disclosure, Armstrong, Vickers, and Zhou (2009) systematically analyze how such prominence may affect the equilibrium and welfare of the search market. One of their findings is that if there are infinitely many firms and the signal distribution is identical for all firms, then prominence has no impact on prices and welfare; it merely redistributes the industry surplus among the firms. We show below that this result continues to hold, even though asymmetric design between the prominent and non-prominent firms is allowed.

Formally, suppose firm 1 is made prominent and so is sampled first by all consumers. Let G_1 be its signal distribution. All other firms are non-prominent. If they are sampled, they are sampled randomly as before. Suppose they share the same signal distribution G. Imagine the situation where the consumers have sampled the prominent firm but rejected its offering. They now face the same situation as they do in the previously analyzed symmetric design case. If G induces an equilibrium with active search, consumers will search these non-prominent firms and obtain surplus $v \in [0, v^*(s)]$.[25] If, instead, G does not induce such an equilibrium, they simply stop searching and obtain $v = 0$. Hence, when consumers are matched with the prominent firm, they simply compare their current surplus $q - p_1$ with their continuation surplus v, where p_1 is the prominent firm's price. If the overall market is active, that is, they search at least the prominent firm, the overall consumer surplus is $-s + \int_0^1 \max\{q - p_1, v\} \mathrm{d}G_1(q) = v - s + c_{G_1}(b_1)$, where $b_1 \equiv v + p_1$. Therefore, this asymmetric design problem can be formulated as choosing an $c_1 \in \mathcal{C}_F$ for the prominent firm and a continuation surplus $v \in [0, v^*(s)]$ for consumers, subject to the prominent firm's incentive constraint:

$$\begin{aligned} &\max_{v, c_1} \; v - s + c_1(b_1) \\ \text{s.t. } & v \in [0, v^*(s)] \text{ and } c_1 \in \mathcal{C}_F, \\ & \exists b_1 \text{ s.t. } -(b_1 - v)c_1'(b_1-) \geq -(x - v)c_1'(x-), \; \forall x \in [v, 1]. \end{aligned} \tag{11}$$

The next proposition tells us that the value of (11) is just $v^*(s)$.

Proposition 6. The optimal consumer surplus under asymmetric design for a prominent firm is still $v^*(s)$. It is achieved by the consumer-optimal symmetric design for all firms.

Therefore, there is no way to make the consumers strictly better off under asymmetric design for a prominent firm than under the simple optimal symmetric design. This implies that the aforementioned finding in Armstrong, Vickers, and Zhou (2009) still holds, even though asymmetry

[25] Using an argument similar to the proof of Corollary 1, we can easily show that the set of achievable consumer surplus is exactly the whole interval $[0, v^*(s)]$.

is allowed. With infinitely many firms, prominence under the optimal design only redistributes the industry surplus between the prominent and non-prominent firms, and it has no impact on the optimal consumer surplus.

6. Concluding remarks

■ This article has investigated the information design problem in a competitive environment where consumers search sequentially for price and product fitness. We constructed a parametric class of signal distributions that generalize the analysis in Roesler and Szentes (2017) to incorporate consumers' search incentives and endogenous outside option, and which is rich enough to achieve every equilibrium consumer surplus that can possibly arise under an arbitrary signal distribution. Relying on the relatively simple parametric nature of this class, we established the existence and uniqueness of the consumer-optimal signal distribution and characterized how the optimal signal distribution changes as with the search friction. We also provided a very simple partial identification for when the information disclosure in a market is not consumer-optimal.

This article only considered consumer-surplus maximization as the information designer's objective. This objective is natural for online rating platforms, as we discussed in the introduction, but there are also other platforms, such as eBay, whose revenues come from the commission fees they charge from the sellers. These platforms would intuitively adopt strategies that favor the sellers and the industry more than the consumers. Then, it is also natural to ask what the industry-optimal information design would be for these markets.

Admittedly, our analysis for the consumer-optimal design is not readily extendable to the industry-optimal design. Although $\mathcal{U}$ is rich enough to achieve every feasible equilibrium consumer surplus, there is no guarantee that this class can achieve the highest possible industry surplus. Intuitively, this is due to the restriction $a \leq v$ that we imposed in the construction of every $c_{a,b,v}$. Although this restriction is important in our analysis to guarantee the firms' downward incentives, it imposes exogenous caps on the equilibrium prices.[26] Therefore, to obtain the industry-optimal design, new signal distributions other than those in $\mathcal{U}$ must be constructed. We leave it for future research.

Appendix: Proof of Proposition 1

For ease of exposition, let $h(x; a, b, v) \equiv s - \frac{(\mu-s-a)(b-v)}{b-a} \log \frac{x-v}{b-v}$. Thus, $c_{a,b,v}(x) = \max\{h(x; a, b, v), 0\}$ for $x \in [b, 1]$.

Proof of Proposition 1. We have explained the first part in the main text. Here, we show the second part. Suppose $c \in \mathcal{C}_F$ induces equilibrium (b, v). Let $a \equiv \mathbb{E}[q|q < b]$, where the expectation is with respect to the signal distribution of c. If $b > \mu - s$, then $a = \frac{\mu-s+bc'(b-)}{1+c'(b-)}$ and is just the x-coordinate of the intersection of c_{F_0} over $[0, \mu]$ and the left tangent line to c at b. See both panels in Figure A1 for an illustration. If $b = \mu - s$, then $a = 0$. Let $\pi \equiv -c'(b-)(b - v)$ be the equilibrium expected profits of a matched firm under c. Note that π can also be written as $\pi = \frac{(\mu-s-a)(b-v)}{b-a}$. We discuss two cases.

First, consider the case $v \geq a$. We show $c_{a,b,v} \leq c$. Panel (a) in Figure A1 provides an illustration of this case. Because $c_{a,b,v}$ coincides with c_{F_0} over $[0, a]$ and is the left tangent line of c at b over $[a, b]$, we know that $c_{a,b,v}(x) \leq c(x)$ for $x \in [0, b]$. Because (b, v) is an equilibrium under c, firms' upward incentive constraints imply

$$c'(x-) \geq -\frac{\pi}{x-v} = -\frac{(\mu-s-a)(b-v)}{(b-a)(x-v)} = h_x(x; a, b, v), \ \forall x \in [b, 1]. \tag{A1}$$

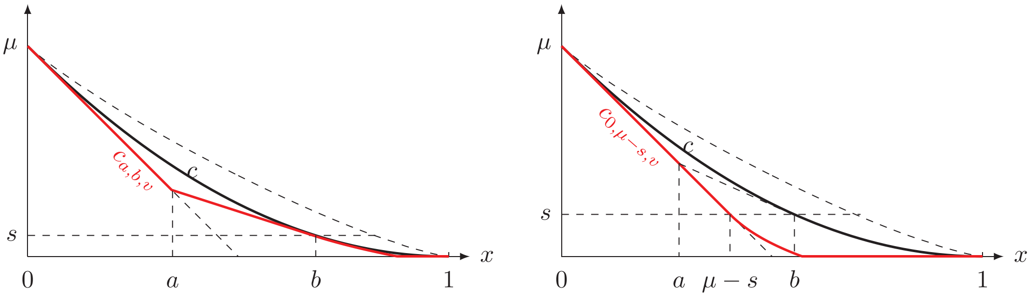
Hence, for $x \in (b, 1]$,

$$h(x; a, b, v) = h(b; a, b, v) + \int_b^x h_x(\tilde{x}; a, b, v)\mathrm{d}\tilde{x} \leq c(b) + \int_b^x c'(\tilde{x}-)\mathrm{d}\tilde{x} = c(x),$$

[26] For example, if the value distribution is uniform and the search cost is $s = 0.125$, then full information disclosure is industry-optimal. This is because its equilibrium achieves the first best total welfare $\bar{b} = 1 - \sqrt{2s} = 0.5$, which in turn is fully extracted by the industry as $p = \sqrt{2s} = 0.5$. If this level of industry surplus was achieved by some $c_{a,b,v} \in \mathcal{U}$, we must have $b = \bar{b}$ and $v = 0$. The restriction $a \leq v$ then requires $a = 0$, but it is straightforward to see that $c_{0,\bar{b},0}$ is not feasible.

FIGURE A1

PROOF OF PROPOSITION 1



(a) If $v \geq a$, $c_{a,b,v}$ is feasible. (b) If $v < a$, $c_{0,\mu-s,v}$ is feasible.

where the inequality comes from (A1) and $h(b; a, b, v) = s = c(b)$. Therefore, $c_{a,b,v}(x) \leq c(x)$ for $x \in (b, 1]$ too.

Next, consider the case $v < a$. We show that $c_{0,\mu-s,v} \leq c$. Panel (b) in Figure A1 provides an illustration. Because $c_{0,\mu-s,v}$ coincides with c_{F_0} over $[0, \mu - s]$, we know that $c_{0,\mu-s,v}(x) \leq c(x)$ for $x \in [0, \mu - s]$. Because $h(x; 0, \mu - s, v) < s \leq c(x)$ for $x \in (\mu - s, b]$, we know that $c_{0,\mu-s,v}(x) \leq c(x)$ for $x \in (\mu - s, b]$. Because $a = \frac{\mu - s + bc'(b-)}{1+c'(b-)}$ and $v < a$, we know that $\pi < \mu - s - v$. Because (b, v) is an equilibrium under c, firms' upward incentive constraints imply

$$c'(x-) \geq -\frac{\pi}{x - v} > -\frac{\mu - s - v}{x - v} = h_x(x; 0, \mu - s, v), \quad \forall x \in (b, 1]. \tag{A2}$$

Hence, for $x \in (b, 1]$,

$$h(x; 0, \mu - s, v) = h(b; 0, \mu - s, v) + \int_b^x h_x(\tilde{x}; 0, \mu - s, v)\mathrm{d}\tilde{x} < c(b) + \int_b^x c'(\tilde{x}-)\mathrm{d}\tilde{x} = c(x),$$

where the inequality comes from (A2) and $h(b; 0, \mu - s, v) < s = c(b)$. Therefore, $c_{0,\mu-s,v}(x) \leq c(x)$ for all $x \in (b, 1]$ too.vv □

References

Anderson, S.P. and Renault, R. "Pricing, Product Diversity, and Search Costs: A Bertrand-Chamberlin-Diamond Model." *The RAND Journal of Economics*, Vol. 30 (1999), pp. 719–735.

Armstrong, M. "Ordered Consumer Search." *Journal of the European Economic Association*, Vol. 15 (2017), pp. 989–1024.

Armstrong, M., Vickers, J., and Zhou, J. "Prominence and Consumer Search." *The RAND Journal of Economics*, Vol. 40 (2009), pp. 209–233.

Armstrong, M. and Zhou, J. "Consumer Information and the Limits to Competition." Working Paper, Cowles Foundation Discussion Papers 2595, 2021.

Au, P.H. and Kawai, K. "Competitive Information Disclosure By Multiple Senders." *Games and Economic Behavior*, Vol. 119 (2020), pp. 56–78.

Au, P.H. and Whitmeyer, M. "Attraction Versus Persuasion." *arXiv e-prints*, pp. arXiv–1802.

Bar-Isaac, H., Caruana, G., and Cuñat, V. "Search, Design, and Market Structure." *American Economic Review*, Vol. 102 (2012), pp. 1140–1160.

Bergemann, D. and Morris, S. "Information Design: A Unified Perspective." *Journal of Economic Literature*, Vol. 57 (2019), pp. 44–95.

Blackwell, D. "Comparison of Experiments." In J. Neyman, ed., *Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability*, pp. 93–102. University of California Press, Berkeley, California, 1951.

Board, S. and Lu, J. "Competitive Information Disclosure in Search Markets." *Journal of Political Economy*, Vol. 126 (2018), pp. 1965–2010.

Choi, M., Dai, A.Y., and Kim, K. "Consumer Search and Price Competition." *Econometrica*, Vol. 86 (2018), pp. 1257–1281.

Choi, M., Kim, K., and Pease, M. "Optimal Information Design for Search Goods." *AEA Papers and Proceedings*, Vol. 109 (2019), pp. 550–556.

Condorelli, D. and Szentes, B. "Information Design in the Holdup Problem." *Journal of Political Economy*, Vol. 128 (2020), pp. 681–709.

Dworczak, P. and Martini, G. "The Simple Economics of Optimal Persuasion." *Journal of Political Economy*, Vol. 127 (2019), pp. 1993–2048.
Gentzkow, M. and Kamenica, E. "A Rothschild-Stiglitz Approach to Bayesian Persuasion." *American Economic Review: Papers & Proceedings*, Vol. 106 (2016), pp. 597–601.
Hewitt, E. and Stromberg, K. *Real and Abstract Analysis: A Modern Treatment of the Theory of Functions of a Real Variable*. Springer-Verlag, New York, 1965.
Hwang, I., Kim, K., and Boleslavsky, R. "Competitive Advertising and Pricing." Working Paper, Emory University and University of Miami, 2019.
Johnson, J.P. and Myatt, D.P. "On the Simple Economics of Advertising, Marketing, and Product Design." *American Economic Review*, Vol. 96 (2006), pp. 756–784.
Kamenica, E. and Gentzkow, M. "Bayesian Persuasion." *American Economic Review*, Vol. 101 (2011), pp. 2590–2615.
Kolotilin, A. "Optimal Information Disclosure: A Linear Programming Approach." *Theoretical Economics*, Vol. 13 (2018), pp. 607–635.
Rayo, L. and Segal, I. "Optimal Information Disclosure." *Journal of Political Economy*, Vol. 118 (2010), pp. 949–987.
Roesler, A.K. and Szentes, B. "Buyer-Optimal Learning and Monopoly Pricing." *American Economic Review*, Vol. 107 (2017), pp. 2072–2080.
Weitzman, M.L. "Optimal Search for the Best Alternative." *Econometrica*, Vol. 47 (1979), pp. 641–654.
Wolinsky, A. "True Monopolistic Competition as a Result of Imperfect Information." *Quarterly Journal of Economics*, Vol. 101 (1986), pp. 493–511.
Zhou, J. "Ordered Search in Differentiated Markets." *International Journal of Industrial Organization*, Vol. 29 (2011), pp. 253–262.

Supporting information

Additional supporting information may be found online in the Supporting Information section at the end of the article.

Figure B.1: Minimal feasible low match value atom
Figure B.2: Illustration of $c_{a^\lambda, b^\lambda, v^\lambda}$ over interval $[a^\lambda, b^\lambda]$
Figure C.1: Proof of Claim C.1
Figure C.2: Illustration of the intervals in Claim C.2
Figure C.3: Proof of part (iii) of Proposition 3
Figure D.1: Graph for Claim D.1
Figure D.2: Change of variables
Figure D.4: Illustration of Claims D.6 to D.8
Figure D.5: Summary of the proof of Claim C.2